Pergamon

J. Appl. Maths Mechs, Vol. 58, No. 2, pp. 335–342, 1994

0021–8928/94 $24.00+0.00

0021–8928(94)E0057-H

THE TEMPERATURE FIELD AND THERMOELASTIC STATE OF A PLATE WITH A PERIODIC SYSTEM OF THIN ELASTIC INCLUSIONS†

V. K. OPANASOVICH, L. O. TISOVSKII and I. I. FEDIK

L'vov, Podol'sk

(*Received* 25 *March* 1993)

The solution of the planar problem of heat conduction and thermoelasticity in the case of a plate with a periodic system of straight thin elastic inclusions of finite length is constructed using the methods of the theory of functions of a complex variable. Integral representations are obtained for the complex temperature and stress–strain state potentials, the system of integro-differential resolvents of the problem is constructed and expressions are presented for the stress intensity factors at the vertices of the inclusions. A numerical analysis of the solution of the problem is carried out using the method of mechanical quadratures.

1. FORMULATION OF THE PROBLEM

An isotropic plate (matrix) containing a system of straight thin-walled elastic inclusions of length $2l$ and thickness $2h$ is considered. The matrix is under the action of a thermal flux at infinity of intensity q_∞. It is assumed that the lateral edges of the plate are thermally insulated and that there is ideal force and thermal contact on the lines of separation of the materials. It is required to determine the effect of the inclusions on the magnitude and character of the temperature-field distribution and to investigate the thermoelastic state in the composite under consideration.

We introduce the system of coordinates $x_1O_1y_1$ (Fig. 1) with the x_1 axis passing through the centres of the inclusions and, also, a local system of coordinates xy, the axes of which are directed along the axes of symmetry of an inclusion. Let α be the angle of inclination of an inclusion to the x_1 axis and d be the distance between the centres of the inclusions. Quantities, referring to inclusions, are denoted by a zero subscript. Since all the inclusions are under identical conditions, the boundary conditions are written for just a single inclusion.

The conditions of mechanical and thermophysical contact of an inclusion with the surrounding material have the form

$$(\sigma_y - i\tau_{xy})^\pm = (\sigma_y - i\tau_{xy})^\pm, \quad \frac{\partial}{\partial x}(u+i\upsilon)_0^\pm + i\varepsilon_0 = \frac{\partial}{\partial x}(u+i\upsilon)^\pm \tag{1.1}$$

$$(T+i\eta)_0^\pm = (T+i\eta)^\pm, \quad k_0\frac{\partial}{\partial y}(T+i\eta)_0^\pm = k\frac{\partial}{\partial y}(T+i\eta)^\pm \tag{1.2}$$

†*Prikl. Mat. Mekh.* Vol. 58, No. 2, pp. 139–146, 1994.

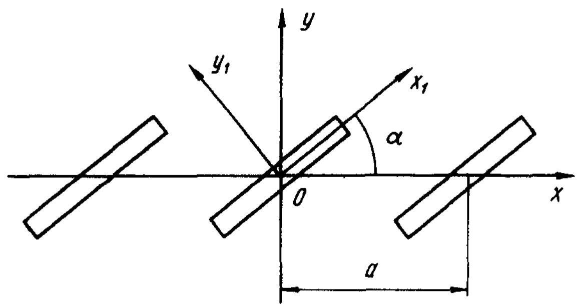

Fig. 1.

The boundary values of functions at the upper edge $(y=+h)$ and the lower edge $(y=-h)$ of an inclusion are denoted by plus and minus signs $\eta(x,y)$ is an auxiliary harmonic function and k_0 and k are the thermal conductivities of the material of the inclusion and the plate respectively.

2. THE HEAT-CONDUCTION PROBLEM

We know [1] that the temperature field in a homogeneous isotropic plate can be defined by the relationships

$$F_1(z)+Q_1(z)=T+i\eta,\quad F(z)+Q(z)=\partial(T+i\eta)/\partial x$$
$$F(z)-Q(z)=-i\partial(T+i\eta)/\partial y,\quad F(z)=F_1'(z),\quad Q(z)=Q_1'(z) \tag{2.1}$$

where $F_1(z)$ and $Q_1(z)$ are the functions that are piecewise holomorphic in the complex plane.

Taking account of the thin-walled nature of an inclusion, we model it with a line provided with specific thermophysical properties. For this purpose, we expand the complex potentials $F_0(z)$ and $Q_0(z)$ in series in the parameter h. Neglecting quantities of a higher order of smallness compared with h, from (2.1) we find

$$x\in L,\ \frac{\partial}{\partial x}(T+i\eta)_0^+-\frac{\partial}{\partial x}(T+i\eta)_0^-=2h\rho'(x),\quad \frac{\partial}{\partial y}(T+i\eta)_0^+-\frac{\partial}{\partial y}(T+i\eta)_0^-=-2hg'(x)$$
$$\frac{\partial}{\partial x}(T+i\eta)_0^++\frac{\partial}{\partial x}(T+i\eta)_0^-=2g(x),\quad \frac{\partial}{\partial y}(T+i\eta)_0^+-\frac{\partial}{\partial y}(T+i\eta)_0^-=2\rho(x) \tag{2.2}$$

where $g(x)$ and $\rho(x)$ are functions which have to be determined and $L=[-l,\ l]$ is a segment of the real axis.

Piecewise holomorphic functions $F(z)$ and $Q(z)$ are introduced in the case of the matrix. Here, the boundary conditions from the edges of an inclusion are brought together on the real axis Ox. On satisfying conditions (1.1) using relationships (2.1) and taking account of the relation (2.2), we obtain the following boundary conditions

$$x\in L,\ [F(x)+Q(x)]^+-[F(x)+Q(x)]^-=2ihk_1g'(x),$$
$$[F(x)-Q(x)]^+-[F(x)-Q(x)]^-=2h\rho'(x) \tag{2.3}$$

$$x\in L,\ [F(x)+Q(x)]^++[F(x)+Q(x)]^-=2g(x),$$
$$[F(x)-Q(x)]^++[F(x)-Q(x)]^-=-2ik_1\rho(x) \tag{2.4}$$

where $k_1 = k_0 / k$.

On solving the linear coupling problems (2.3), we find

$$F(z_1) = \frac{h}{2d} \int_{-l}^{l} \operatorname{ctg}\left(\frac{\pi}{d}(te^{i\alpha} - z_1)\right)[k_1 g'(t) - i\rho'(t)]\,dt + c$$

$$Q(z_1) = \frac{h}{2d} \int_{-l}^{l} \operatorname{ctg}\left(\frac{\pi}{d}(te^{i\alpha} - z_1)\right)[k_1 g'(t) + i\rho'(t)]\,dt + c \tag{2.5}$$

$$c = -q_\infty e^{-i\varphi}(2k)^{-1}, \quad z_1 = x_1 + iy_1$$

where φ is the angle between the Ox axis and the principal direction of the thermal flux at infinity of intensity q_∞.

If, now, we satisfy conditions (2.4), taking account of (2.5), we obtain a system of integro-differential equations for determining the unknown functions $g(x)$ and $\rho(x)$

$$x \in L,\ g(x) - \frac{h}{\pi} \int_{-l}^{l} [k_1 g'(t)\, K(t-x) + \rho'(t)\, L(t-x)]\,dt = -aq_\infty k^{-1} \cos(\alpha - \varphi) \tag{2.6}$$

$$k_1 \rho(x) + \frac{h}{\pi} \int_{-l}^{l} [k_1 g'(t)\, L(t-x) - \rho'(t)\, K(t-x)]\,dt = bq_\infty k^{-1} \sin(\alpha - \varphi)$$

Here

$$a = 1 - \tilde{\varepsilon},\ b = 1 - k_1\tilde{\varepsilon}, \tilde{\varepsilon} = \min(1, k_1^{-1}),\ K(x) + iL(x) = Q\operatorname{ctg}(Qx),\ Q = \pi e^{i\alpha} d^{-1}$$

To Eqs (2.6), it is necessary to add the relationships

$$\int_{-l}^{l} g'(t)\,dt = 0, \quad \int_{-l}^{l} \rho'(t)\,dt = 0 \tag{2.7}$$

which are the conditions for the temperature and the heat balance to be unique on passing around the contour of an inclusion.

We shall seek a solution of the system of equations (2.6) and (2.7) in the form

$$g'(x) = Z(x) / \sqrt{1-x^2}, \quad \rho'(x) = Y(x) / \sqrt{1-x^2} \tag{2.8}$$

Using the method of mechanical quadratures [2], we arrive at the system of linear algebraic equations for finding the value of the unknown functions $Z(x)$ and $Y(x)$ at the nodal points

$$\sum_{m=1}^{M} \{\pi Z(t_m)\, A(m,r) - h[k_1 Z(t_m)\, K(l(t_m - x_r)) + Y(t_m)\, L(l(t_m - x_r))]\} = -Maq_\infty k^{-1} \cos(\alpha - \varphi)$$

$$\sum_{m=1}^{M} \{\pi k_1 Y(t_m)\, A(m,r) + h[k_1 Z(t_m)\, L(l(t_m - x_r)) - Y(t_m)\, K(l(t_m - x_r))]\} =$$

$$= Mbq_\infty k^{-1} \sin(\alpha - \varphi) \tag{2.9}$$

Here

$$A(m,r) = -\frac{2}{\pi} \sum_{k=1}^{M-1} \frac{1}{k} T_k(t_m)\, U_{k-1}(x_r) \sqrt{1 - x_r^2}$$

$$t_m = \cos\frac{2m-1}{2M}\pi, \quad m = 1, 2, \ldots, M; \quad x_r = \cos\frac{\pi r}{M}, \quad r = 1, 2, \ldots, M-1$$

$$U_{m-1}(x) = \frac{\sin(n \arcsin x)}{\sqrt{1-x^2}}, \quad T_n(x) = \cos(n \arccos x)$$

Having a solution of the system of algebraic equations (2.9), it is possible to construct an interpolating Lagrange polynomial for the functions $Y(x)$ and $Z(x)$ [3] using the Chebyshev nodal points

$$Y(x)=\sum_{r=1}^{M-1} y_r T_r(x),\quad Z(x)=\sum_{r=1}^{M-1} z_r T_r(x) \tag{2.10}$$

$$\left(y_r=\frac{2}{M}\sum_{m=1}^{M} Y(t_m)\,T_r(t_m),\quad z_r=\frac{2}{M}\sum_{m=1}^{M} Z(t_m)\,T_r(t_m)\right)$$

By substituting the expressions for the functions $g'(x)$ and $\rho'(x)$ (2.8) into (2.5) and taking account of (2.10), the complex potentials $F(z_1)$ and $Q(z_1)$ can be represented in the form

$$F(z_1)=B\sum_{r=1}^{M-1}\sum_{k=-\infty}^{\infty}(k_1 z_r - iy_r)\,L_r(\upsilon_k)+c$$
$$Q(z_1)=B\sum_{r=1}^{M-1}\sum_{k=-\infty}^{\infty}(k_1 z_r + iy_r)\,L_r(\upsilon_k)+\bar{c} \tag{2.11}$$

Here

$$B=he^{-i\alpha}/(2\pi l),\quad \upsilon_k=(z_1-\pi dk)\,e^{-i\alpha}l^{-1},\quad u_k=e^{2i\alpha_k}\upsilon_k$$
$$L_r(z)=[U_{r-1}(z)\sqrt{z^2-1}-T_r(z)]/\sqrt{z^2-1}$$

Note that, if one puts $k_0=0$ in (2.11), we obtain the solution of the heat-conduction problem for a plate with a periodic system of thermally insulated cracks [5]. If, however, one puts $k_0=k$, we obtain the solution of the heat-conduction problem for a plate without inclusions.

3. THERMOELASTICITY PROBLEMS

The thermoelastic state of an isotropic plate can be described using the complex potentials $\Phi(z)$ and $\Psi(z)$, starting out from the formulae [1]

$$\sigma_y+\sigma_x=2[\Phi(z)+\overline{\Phi(z)}],\quad \sigma_y-i\tau_{xy}=\Phi(z)+R(z) \tag{3.1}$$
$$2\mu\frac{\partial}{\partial x}(u+i\upsilon)=\kappa\Phi(z)-R(z)+H\Psi_1(z)$$

Here

$$\Psi_1(z)=\frac{1}{2}\int[F(z)+\overline{Q}(z)]\,dz$$
$$R(z)=\overline{\Phi(z)}+z\overline{\Phi'(z)}+\overline{\Psi(z)},\quad \overline{Q}(z)=\overline{Q(\bar{z})}$$

$\kappa=3-4\nu$, $H=2\alpha E$ in the case of plane strain $\kappa=(3-\nu)/(1+\nu)$, $H=2\alpha E/(1+\nu)$ for the plane stressed state α is the temperature coefficient of linear expansion, ν is Poisson's ratio, E is Young's modulus and μ is the Lamé coefficient.

Taking account of the thin-walled nature of an inclusion, let us expand the complex potentials $\Phi_0(z)$, $\Psi_0(z)$ and $\Psi_{01}(z)$ in relationships (3.1) in a Taylor series in the neighbourhood of a point x on the real axis.

On retaining terms of an order not higher than h in the final expansions, we obtain the relations

$$x\in L,\ (\sigma_y-i\tau_{xy})_0^+-(\sigma_y-i\tau_{xy})_0^-=2ihK'(x)$$
$$\frac{\partial}{\partial x}(u+i\upsilon)_0^+-\frac{\partial}{\partial x}(u+i\upsilon)_0^-=ih[M'(x)+H_0\Psi_0'(x)]/\mu_0$$
$$(\sigma_y-i\tau_{xy})_0^++(\sigma_y-i\tau_{xy})_0^-=2\gamma_0[(1-\kappa_0)\,K(x)+2M(x)+2\overline{K(x)}+2\overline{M(x)}] \tag{3.2}$$

$$\frac{\partial}{\partial x}(u+iv)_0^+ + \frac{\partial}{\partial x}(u+iv)_0^- = \mu_0^{-1}\gamma_0[2\kappa_0 K(x)+(\kappa_0-1)M(x)-2\overline{K(x)}-2\overline{M(x)}]+H_0\Psi_0(x)/\mu_0$$

Here

$$\Psi_0(x)=\frac{1}{2}T_0-\tilde{\varepsilon}\tilde{c}x+\frac{1}{4}\Big\{(z_1-iy_1)\left[-\frac{x}{l}\sqrt{l^2-x^2}+l\arcsin\frac{x}{l}\right]+$$
$$+\sum_{r=2}^{M-1}\frac{1}{r}(z_r-iy_r)\sqrt{l^2-x^2}\left[\frac{1}{r-1}U_{r-2}\left(\frac{x}{l}\right)-\frac{1}{r+1}U_r\left(\frac{x}{l}\right)\right]\Big\}$$
$$\gamma_0=(1+\kappa_0),\quad \tilde{c}=ce^{i\alpha}$$

where T_0 is the temperature of the inclusion and $K(x)$ and $M(x)$ are unknown functions.

Relations (3.2) provide a model of a thin inclusion for the planar problem of thermoelasticity.

Making use of the conditions of ideal mechanical contact (1.1) and, also, relations (3.2) and (3.1) for determining the complex potentials $\Phi(z)$ and $\Psi(z)$, we shall have the following boundary-value problems

$$x\in L,[\Phi(x)+R(x)]^+-[\Phi(x)+R(x)]^-=2ihK'(x)$$
$$[\kappa\Phi(x)-R(x)+H\Psi_1(x)]^+-[\kappa\Phi(x)-R(x)+H\Psi_1(x)]^-=$$
$$=2ih\beta_0[M'(x)-M_1^*(x)+H_0\Psi_0'(x)] \tag{3.3}$$
$$[\Phi(x)+R(x)]^++[\Phi(x)+R(x)]^-=2\gamma_0[(1-\kappa_0)K(x)+2\overline{K(x)}+2M(x)+2\overline{M(x)}]$$

$$[\kappa\Phi(x)-R(x)+H\Psi_1(x)]^++[\kappa\Phi(x)-R(x)+H\Psi_1(x)]^-=$$
$$=2\beta_0\gamma_0[2\kappa_0K(x)+(\kappa_0-1)M(x)-2\overline{K(x)}-2\overline{M(x)}]+2\beta_0H_0\Psi_0(x)-2i\varepsilon_0^* \tag{3.4}$$

Here

$$\varepsilon_0^*=-2\mu\varepsilon_0,\quad M_1^*=\omega_0H\tilde{c},\quad \beta_0=\mu/\mu_0,\quad \omega_0=\min(1,\beta_0^{-1})$$
$$\Psi_1(z_1)=c_1z_1+\frac{1}{2}T_\infty+\Psi_1^*(z_1)=c_1z_1+\frac{1}{2}T_\infty+\frac{h}{2}\sum_{r=1}^{M-1}\sum_{k=-\infty}^{\infty}\frac{(k_1z_r-iy_r)l^r}{(\tilde{z}+\sqrt{\tilde{z}^2-l^2})^r}$$
$$\tilde{z}=e^{-i\alpha}(z_1-\pi dk)$$

and T_∞ is the value of the temperature at infinity.

We satisfy conditions (3.3), if the complex potentials are represented in the form

$$\Phi(z_1)=\gamma_2\int_{-l}^{l}[K_0'(t)+\beta_0M_0'(t)]\operatorname{ctg}[P(t,z_1)]\,dt-\gamma_3\Psi_1^*(z)$$
$$\Psi(z_1)=\gamma_2\int_{-l}^{l}\{[\beta_0M_0'(t)]-\kappa K_0'(t)]\operatorname{ctg}[P(t,z_1)]e^{-i\alpha}- \tag{3.5}$$
$$-\{\pi d^{-1}[z_1d^\alpha+t(1-e^{2i\alpha})]\operatorname{cosec}^2[P(t,z_1)]+e^{i\alpha}\operatorname{ctg}[P(t,z_1)]\}\times$$
$$\times[K_0'(t)+\beta_0M_0'(t)]\}\,dt-\gamma_3he^{-i\alpha}(2d)^{-1}\int_{-l}^{l}\{[k_1g_0(t)+$$
$$+i\rho_0(t)-e^{2i\alpha}(k_1g_0(t)-i\rho_0(t))]\operatorname{ctg}[P(t,z_1)]-[k_1g_0(t)-$$
$$-i\rho_0(t)](te^{-i\alpha}-te^{i\alpha}+z)\pi d^{-1}e^{2i\alpha}\operatorname{cosec}^2[P(t,z_1)]\}\,dt$$
$$\gamma=[\pi(1+\kappa)]^{-1},\quad K_0(x)=K(x),\quad M_0'(x)=M'(x)-M_1^*(x)+H_0\Psi_0'(x),$$
$$\gamma_2=\gamma\pi d^{-1}e^{i\alpha},\quad \gamma_3=\pi\gamma H,\quad P(t,z_1)=\pi d^{-1}(te^{i\alpha}-z_1)$$

On substituting the expressions for the functions $\Phi(z)$ and $\Psi(z)$ into relationships (3.4), we arrive at a system of integro-differential equations in the unknown functions $K_0(x)$ and $M_0(x)$

$$
\begin{aligned}
& x\in L,\quad \gamma_0[(1-\kappa_0)K_0(x)+2M_0(x)+2\overline{K_0(x)}+2\overline{M_0(x)}]-\\
& -c_1\int_{-l}^{l}\{[K_0'(t)+\beta_0 M_0'(t)]\,g(t-x)+[-\kappa K_0'(t)+\beta_0 M_0'(t)]\,\overline{g(t-x)}+\\
& +[\overline{K_0'(t)}+\beta_0\overline{M_0'(t)}]\,G(t-x)\}\,dt=A(x) \qquad (3.6)\\
& \beta_0\gamma_0[2\kappa_0 K_0(x)+(\kappa_0-1)M_0(x)-2\overline{K_0(x)}-2\overline{M_0(x)}]+\\
& +c_1\int_{-l}^{l}\{-\kappa[K_0'(t)+\beta_0 M_0'(t)]\,g(t-x)+[-\kappa[K_0'(t)+\beta_0 M_0'(t)]\overline{g(t-x)}+\\
& +[\overline{K_0'(t)}+\beta_0\overline{M_0'(t)}]\,G(t-x)\}\,dt=B(x)+i\varepsilon_0^*
\end{aligned}
$$

Here

$$c_1=h\gamma,\quad \lambda=d/l,\quad q=-i\lambda e^{-i\alpha}\sin\alpha,\quad g(x)=Q\mathrm{ctg}(Qx)$$

$$G(x)=\pi d^{-1}(e^{-i\alpha}-e^{-3i\alpha})\,[\mathrm{ctg}(Qx)-Qx\,\mathrm{cosec}^2(Qx)]$$

$$A(x)=-4\gamma_0\,\mathrm{Re}\left[H\omega_0\left(\frac{1}{2}T_0+\tilde{c}x\right)-H\Psi_0(x)\right]-C(x)$$

$$
\begin{aligned}
B(x)=&-\beta_0\gamma_0\left\{(\kappa_0-1)\left[H\omega_0\left(\frac{1}{2}T_0+\tilde{c}x\right)-H_0\Psi_0(x)\right]+\right.\\
&\left.+2\left[H_0\overline{\Psi_0(x)}-H\omega_0\left(\frac{1}{2}T_0+\tilde{c}x\right)\right]\right\}+C(x)+H\left(\frac{1}{2}T_0+\tilde{c}x\right)-\beta_0 H_0\Psi_0(x)
\end{aligned}
$$

$$C(x)=c_1 H\pi\sum_{r=1}^{M-1}\left\{(k_1 z_r-iy_r)\,\mathrm{Re}\left[P_r\left(\frac{x}{l}\right)\right]+(k_1 z_r+iy_r)\,S_r\left(\frac{x}{l}\right)\right\}$$

$$P_r(x)=\frac{1}{r}\left\{T_r(x)+\sum_{k=1}^{\infty}\left[\frac{1}{(x+\lambda ke^{-i\alpha}+R^+e^{i\omega^+})^r}+\ldots\right]\right\}$$

$$S_r(x)=q\sum_{k=1}^{\infty}k\left[\frac{e^{i\omega^+}}{R^+(x+\lambda ke^{i\alpha}+R^+e^{-i\omega^+})^r}-\ldots\right]$$

$$R^\pm=\{[(\Delta^\mp-1)^2+\delta^2]\,[(\Delta^\mp+1)^2+\delta^2]\}^{1/4}$$

$$2\omega^+=\begin{cases}2\pi-\gamma^--\gamma^+, & \Delta^-+1<0\\ \pi-\gamma^-+\gamma^+, & \Delta^-+1>0\end{cases},\qquad 2\omega^-=\begin{cases}4\pi-\beta^--\beta^+, & \Delta^--1>0\\ 3\pi+\beta^--\beta^+, & \Delta^--1<0\end{cases}$$

$$\beta^\pm=\mathrm{arctg}\frac{\delta}{|\Delta^+\pm 1|},\quad \gamma^\pm=\mathrm{arctg}\frac{\delta}{|\Delta^-\pm 1|}$$

$$\Delta^\pm=x\pm\lambda k\cos\alpha,\quad \delta=\lambda k\sin\alpha$$

$$\Delta^\pm=x\pm\lambda k\cos\alpha,\quad \delta=\lambda k\sin\alpha$$

(the dots in the square brackets, as previously, denote a term in which the plus superscript has been replaced by a minus). The following relationships

$$\int_{-l}^{l}K_0'(t)\,dt=0,\quad \int_{-l}^{l}M_0'(t)\,dt=0,\quad \mathrm{Im}\int_{-l}^{l}tK_0'(t)\,dt=0$$

which are the conditions for the equilibrium of an inclusion and the conditions for the uniqueness of the displacements on passing around the contour of the inclusion, have to be added to the system of equations (3.6).

Having passed to the corresponding limit in (3.6), we obtain the singular integral equations for a plate with a periodic system of cracks [5] and inelastic inclusions.

The asymptotic form of the stress–strain state in the neighbourhood of the vertices of an inclusion have been given in [4] and the stress intensity factors (SIF) are determined using the formulae

$$K_1^{\pm} - iK_2^{\pm} = \mp A\beta_0 \lim_{x\to\pm l} \sqrt{l^2 - x^2}\, M_0'(x)$$

$$K_3^{\pm} - iK_4^{\pm} = \mp A \lim_{x\to\pm l} \sqrt{l^2 - x^2}\, K_0'(x) \quad (A = 2h\pi/(\sqrt{l}(1+\kappa)))$$

We will seek the solution of the system of equations (3.6) in the form

$$K_0'(lx) = u(x)/\sqrt{1-x^2}, \quad M_0'(lx) = \upsilon(x)/\sqrt{1-x^2}$$

Here, $u(x)$ and $\upsilon(x)$ are unknown functions, the values of which at the nodal points are determined from the system of linear algebraic equations obtained by the method of mechanical quadratures [2].

4. NUMERICAL ANALYSIS

Numerical investigations of the generalized SIF at the vertices of thin inclusions as a function of different geometrical and thermophysical parameters of the problem were carried out for the case when a constant temperature T_0 is maintained in a plate with a periodic system of thin linear inclusions. The results are shown in Figs 2–4.

Graphs of the dimensionless SIF $K_i' = K_i/(HT_0\sqrt{l})$ $(i=1, 3)$ as a function of the relative stiffness of a thin inclusion for various values of the parameter $\alpha^* = \alpha_0/\alpha$ and the dimensionless distance between the centres of neighbouring inclusions d/l are shown in Figs 2 and 3. The results for K_3' are represented by the solid lines while those for K_1' are represented by the dashed lines. Note that, in the case under consideration, stresses and displacements in the composite occur not on account of the perturbation of the temperature field but as a result of the differences in the coefficients of linear thermal expansion of the materials of the matrix and the inclusions. Under such conditions $K_2' = K_4' = 0$.

Curves 1, 4 and 7 in Fig. 2 are drawn for a value of the parameter $\alpha^* = 0$, curves 2, 5 and 8 for $\alpha^* = 0.1$ and curves 3, 6 and 9 for $\alpha^* = 0.5$. Lines 1–3 correspond to a relative distance between inclusions $d/l = 25$ (actually, we have the case of a single inclusion in an unbounded plate and these results are identical to those presented in [4]). Lines 4–6 are drawn for $d/l = 3.0$ and lines 7–9 for $d/l = 2.5$.

Results for $\alpha^* \geqslant 1$ are shown in Fig. 3. The even curves are drawn for a value $\alpha^* = 2$ and the odd curves for $\alpha^* = 3$. Note that, when $\alpha^* = 1$, all of the SIF are equal to zero, that is, there is no perturbation of the stressed state in the neighbourhood of the vertices of the inclusions. Graphs 1 and 2 are drawn for $d/l = 2.5$, graphs 3 and 4 for $d/l = 3.0$ and graphs 5 and 6 for $d/l = 2.5$.

The nature of the change in the generalized SIF K_i' as a function of the angle of orientation of an inclusion is shown by the solid curves in Fig. 4 for $\alpha^* = 0$, and by the dashed curves for $\alpha^* = 0.5$. Note that, when $\alpha^* = 2$, the required relations are obtained from the solid curves in Fig. 4 by a symmetric reflection in the abscissa axis. Here, the following notation for the curves has been introduced. Curve 1 represents the behaviour of K_3' for an absolute rigid inclusion. Curves 2 and 3, respectively, give the values of K_3', K_1' for an elastic inclusion with a relative stiffness $\mu_0/\mu = 10$ and curve 4 represents the nature of the change in K_1' for an inclusion with a relative stiffness of 0.1. Note that, in the case under consideration, the remaining quantities are small compared with those which have been presented and are therefore not shown in the figures.

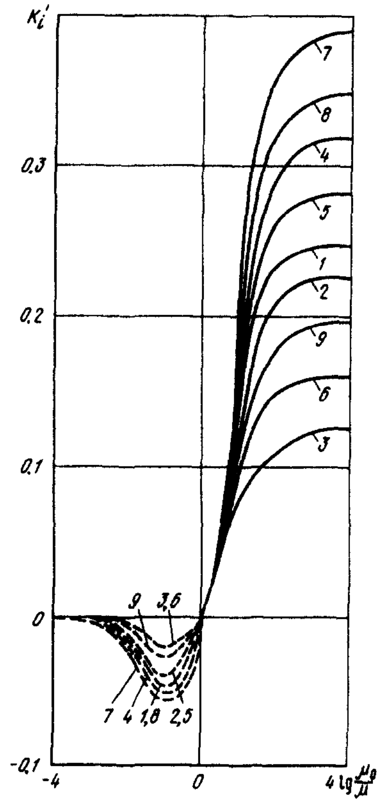

Fig. 2.

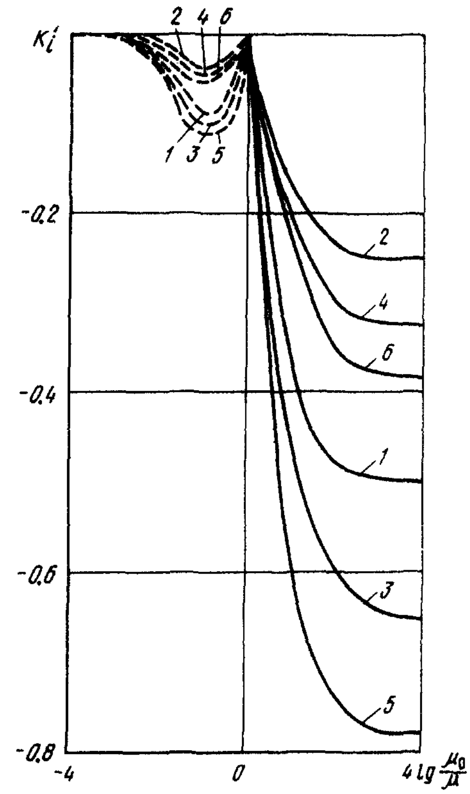

Fig. 3.

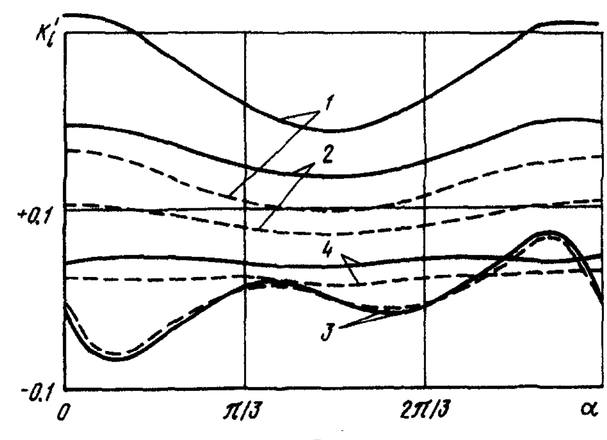

Fig. 4.

REFERENCES

1. PRUSOV I. A., *Some Problems of Thermoelasticity*. Izd. Belorus. Univ., Minsk, 1972.
2. PANASYUK V. V., SAVRUK M. P. and DATSISHIN A. P., *Distribution of Stresses around Cracks in Plates and Shells*. Naukova Dumka, Kiev, 1976.
3. KORN G. A. and KORN T. M., *Mathematical Handbook for Scientists and Engineers*. McGraw-Hill, New York, 1968.
4. GRILITSKII D. V., DRAGAN M. S. and OPANASOVICH V. K., The temperature field and thermoelastic state of a plate with a thin-walled elastic inclusion. *Prikl. Mat. Mekh.* **44**, 2, 338–345, 1980.
5. KIM G. S. and KRIVTSUN M. G., *Planar Problems of Thermoelasticity for Bodies with Cracks*. Naukova Dumka, Kiev, 1983.

Translated by E.L.S.